Optimal Asymmetric Resource Allocation for Dual-hop Multi-relay Based Downlink OFDMA Systems

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Abstract

We proposed an optimal asymmetric resource allocation (ARA) scheme for the decode-and-forward (DF) dual-hop multi-relay systems in the downlink, with Orthogonal Frequency-Division Multiple Access (OFDMA) for multi-user transmission. Our work is different in that the time slots for the two hops via each of the relays are designed to be asymmetric, *i.e.*, with K relays in the cell, a total of $2K$ time slots may be of different durations, which enhances the degree of freedom over the previous work. Also, a destination may be served by multiple relays at the same time to enhance the transmission diversity. Moreover, closed-form results for optimal resource allocation are derived, which require only limited amount of feedback information. Simulation results show that, thanks to the multi-time and multi-relay diversities, the proposed ARA scheme can provide a much better performance than the scheme with symmetric time allocation, as well as the scheme with asymmetric time allocation for a cell composed of independent single-relay sub-systems, especially when the relays are relatively close to the source.

Keywords

Relay; Resource Allocation; OFDMA; Asymmetric Time Allocation; Multi-relay Diversity; Multi-user Diversity

Introduction

Recently, relay technologies, including amplify-and-forward (AF) [Yang08] and decode-and-forward (DF) [Kim08] modes, have been considered as a promising and achievable solution for the fourth-generation (4G) mobile communication systems such as Long Term Evolution Advanced (LTE-Advanced) systems [Yang09] [Iwamura10].

Adaptive resource allocation (RA) [Shen05] plays an essential part in relay systems. In [Li08], subcarrier pairing and power allocation was conducted successively in a single-relay system for both AF and DF modes to maximise the system capacity. In [Wang08], the optimal joint subcarrier matching and power allocation was proposed for DF relay systems with single destination. However, it requires the bits to be grouped in relaying, moreover, the complexity of subcarrier pairing is too high to guarantee that the channel state remains the same during this process. In [Nam07] an optimal RA algorithm was proposed to maximise the bandwidth efficiency of the DF relay system, however, it assumes fixed geometric locations of the relays, which is merely practical in the real urban environment. While in [Salem10], the authors considered a multi-relay system with random relay locations, and also proposed an optimal RA algorithm. However, in most previous work, the time slot durations for the two hops were designed to be symmetric. In [Agustin09], a two-way relay channel model was investigated, and an asymmetric time allocation scheme was proposed, where the two consecutive time slot durations were designed to be asymmetric. However, it is only assumed that a uniform power allocation is conducted across all subcarriers, which was not efficient. In [Zhou11], an optimal ARA scheme was proposed for an OFDM based DF relay system, however, it can only be applied to a scenario where a cell was divided into multiple single-relay sub-systems with RA conducted in each of individual sub-systems independently. Hence, the ARA scheme is not readily extendable to a multi-relay multi-destination system.

In this paper, we investigate an ARA scheme for DF dual-hop multi-relay OFDMA systems. Our work is different in the following aspects. Firstly, to the best of

our knowledge, this is the first work to apply asymmetric time allocation to a general multi-relay multi-destination system, where the time slots for the two hops via each of the relays are designed to be asymmetric, *i.e.*, with *K* relays in the cell, a total of 2*K* time slots may be of different durations. As a result, it enhances the degree of freedom for transmission over the previous symmetric RA (SRA) scheme. Secondly, an optimal algorithm is proposed to perform joint time, power and subcarrier allocation to obtain the global optimal results, with only limited amount of feedback information from relays and destinations. This is not an easy extension of [Zhou11] where each destination is served by only one relay in an independent sub-system. While the proposed work allows multiple relays to serve a single destination, which enhances the degrees of freedom. Simulation results show that, thanks to the multi-time and multi-relay diversities, the proposed ARA scheme outperforms the SRA algorithm in [Salem10], as well as the ARA algorithm in [Zhou11], with higher achievable system throughput, especially when the relays are relatively close to the source. Moreover, we demonstrate the impact of the relays' locations on the results of asymmetric time allocation.

Section II presents the system model and the problem formulation. In Section III, the optimal ARA algorithm is proposed. Simulation results are shown in Section IV, and the conclusion is drawn in Section V.

System Model and Problem Formulation

System Model

We consider a DF OFDMA cellular system in the downlink, with two hops and *K* relays, as illustrated in FIG. 1, where *S* denotes the source node (BS), $D_l\ (l=1,...,L)$ denotes the different destination nodes (mobile users), and $R_k\ (k=1,...,K)$ denotes the multiple relay nodes. The *K* relays are uniformly distributed within the range of width w in FIG. 1. We define d_{SR} as the distance from the source to the mid circle of relays' band. Without the loss of generality, it is assumed that all the destinations have the approximately same distance from the source, which is denoted by d_{SD}, and they are uniformly distributed along a circle. The hop from source to relays is defined as the first hop, and the hop from the relays to destinations is the second hop. We define $\{1,...,N\}$ as the orthogonal subcarriers set, where *N* is the total number of subcarriers.

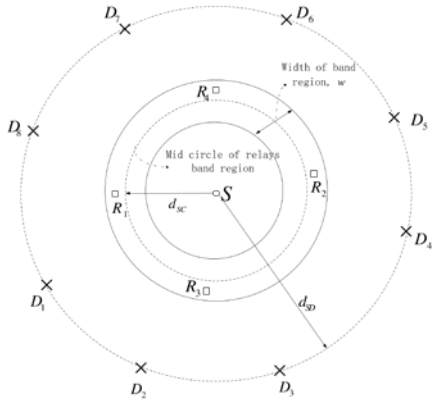


FIG. 1 DUAL-HOP MULTI-RELAY CELLULAR SYSTEM (S - SOURCE; R - THE k TH RELAY ($k=1,...,K$); D - THE l TH DESTINATION ($l=1,...,L$))

The total transmits time duration is denoted as *T*. The time slot durations via the relay *k* for the first and the second hops are $T_{1,k}$ and $T_{2,k}$, respectively. Assuming perfect channel estimation at both relay nodes and destinations, we define $h_{1,k}^{(n)}$ as the normalised channel impulse response on subcarrier *n* between the sources and relay *k* in the first hop, and $h_{2,k,l}^{(n)}$ as the normalised channel impulse response between relay *k* and destination *l* in the second hop. The path loss (PL) attenuation coefficients in the first and the second hops are denoted as A_1 and A_2, respectively. We define d_k as the distance between the sources and relay *k* in the first hop, and $d_{k,l}$ as the distance between the relay *k* to the destination *l* in the second hop. The PL exponents in the first and second hops are denoted as τ_1 and τ_2, respectively. We define $p_{1,k}^{(n)}$ as the transmit power for the source to relay *k* on subcarrier *n* in the first hop, and $p_{2,k,l}^{(n)}$ as the transmit power between relay *k* and destination *l* in the second hop. $\xi_{1,k}^{(n)} \in \{0,1\}$ denotes subcarrier allocation indicator for subcarrier *n* in the first hop. If subcarrier *n* is allocated to relay *k*, $\xi_{1,k}^{(n)}=1$, otherwise, $\xi_{1,k}^{(n)}=0$. Similarly, $\xi_{2,k,l}^{(n)}$ is the subcarrier allocation indicator in the second hop.

The channel-to-noise ratio (CNR) is $\gamma_{1,k}^{(n)} = N\left|h_{1,k}^{(n)}\right|^2 \Big/ \left(A_1 N_0 B d_k^{\tau_1}\right)$ in the first hop, where N_0 is the single-sided power spectral density of the additive white Gaussian noise (AWGN). The

bandwidth for each subcarrier is B/N. In the second hop, we define $\gamma_{2,k,l}^{(n)} = N\left|h_{2,k,l}^{(n)}\right|^2 \Big/ \left(A_2 N_0 B d_{k,l}^{\tau_2}\right)$ as the CNR. The maximum achievable throughput (in bits per second) for relay *k* in the first hop can be written as:

$$C_{1,k} = \frac{B}{N}\sum_{n=1}^{N}\xi_{1,k}^{(n)}\log_2\left(1+p_{1,k}^{(n)}\gamma_{1,k}^{(n)}\right) \quad (1)$$

and the maximum achievable throughput of the second hop can be written as:

$$C_{2,k} = \frac{B}{N}\sum_{l=1}^{L}\sum_{n=1}^{N}\xi_{2,k,l}^{(n)}\log_2\left(1+p_{2,k,l}^{(n)}\gamma_{2,k,l}^{(n)}\right) \quad (2)$$

Hence, the overall maximum achievable throughput during time slot T on the route via relay k can be written as:

$$TC_k = \min\left\{T_{1,k}C_{1,k}, T_{2,k}C_{2,k}\right\} \quad (3)$$

Problem Formulation

In this subsection, the RA problem of the investigated system is formulated. We define P_T as the total transmit power of the system in the downlink, P_S as the transmit power of the source node, and P_R as the total transmit power of all the relay nodes. We also assume that each subcarrier is occupied by only one node in a time slot. Unlike SRA in [Salem10], where the time slot durations for the two hops are identical, the proposed ARA scheme allows all the 2*K* time slots to be of different durations. Also, it allows a certain subcarrier to be allocated to any of the *K* relays in the first hop, while in [Boyd04], *N* subcarriers are divided into *K* groups and each group serves a single-relay sub-system. In the second hop, our system allows multiple relays to serve one destination, which is different from the multiple single-relay sub-systems, where each destination can only be served by the single relay. We maximise the achievable system throughput, which is formulated by:

$$\max_{p_{1,k}^{(n)}, p_{2,k,l}^{(n)}, T_{1,k}, T_{2,k}} \sum_{k=1}^{K} TC_k \quad (4)$$

subject to:

$$T_{1,k} + T_{2,k} = T, \quad (\forall k) \quad (5)$$

$$\sum_{k=1}^{K}\sum_{n=1}^{N} p_{1,k}^{(n)} \le P_S, \quad \sum_{k=1}^{K}\sum_{l=1}^{L}\sum_{n=1}^{N} p_{2,k,l}^{(n)} \le P_R, \quad (6)$$

$$\text{where} \quad p_{1,k}^{(n)} \ge 0, \quad p_{2,k,l}^{(n)} \ge 0, \quad (\forall k,l,n) \qquad P_S + P_R = P_T \quad (7)$$

$$\sum_{k=1}^{K}\xi_{1,k}^{(n)} = 1, \quad \sum_{k=1}^{K}\sum_{l=1}^{L}\xi_{2,k,l}^{(n)} = 1, \quad (\forall n) \quad (8)$$

Constraint (5) implies that the time slot durations for both hops via each relay may be asymmetric, while in [Salem10], they are assumed to be equal, *i.e.*, $T_{1,k} = T_{2,k} = T/2 \,\forall\,(n)$. Constraint (8) indicates the exclusivity of subcarriers allocated to nodes.

Optimal Asymmetric Resource Allocation

In this section, we propose the optimal solution to the objective function (4). It can be derived that (4) is optimised when $T_{1,k}C_{1,k} = T_{2,k}C_{2,k}\,(\forall k)$ [Zhou11]. Also using (3), (4) can be rewritten as:

$$\max_{p_{1,k}^{(n)}, p_{2,k,l}^{(n)}, T_{1,k}, T_{2,k}} \sum_{k=1}^{K}\frac{1}{2}\left(T_{1,k}C_{1,k} + T_{2,k}C_{2,k}\right) \quad (9)$$

It can be shown in the Appendix that (9) is a convex function. Thus (9) can be solved using the Karush-Kuhn-Tucker (KKT) conditions [Boyd04]. Using (9), and (5)-(8), the Lagrangian function can be built as equation:

$$\begin{aligned} L = &\sum_{k=1}^{K}\frac{1}{2}\Bigg[T_{1,k}\sum_{n=1}^{N}\xi_{1,k}^{(n)}\log_2\left(1+p_{1,k}^{(n)}\gamma_{1,k}^{(n)}\right) \\ &+T_{2,k}\sum_{l=1}^{L}\sum_{n=1}^{N}\xi_{2,k,l}^{(n)}\log_2\left(1+p_{2,k,l}^{(n)}\gamma_{2,k,l}^{(n)}\right)\Bigg] \\ &-\sum_{k=1}^{K}\lambda_k\Bigg[T_{1,k}\sum_{n=1}^{N}\xi_{1,k}^{(n)}\log_2\left(1+p_{1,k}^{(n)}\gamma_{1,k}^{(n)}\right) \\ &-T_{2,k}\sum_{l=1}^{L}\sum_{n=1}^{N}\xi_{2,k,l}^{(n)}\log_2\left(1+p_{2,k,l}^{(n)}\gamma_{2,k,l}^{(n)}\right)\Bigg] \\ &-\mu_1\left(\sum_{k=1}^{K}\sum_{n=1}^{N}p_{1,k}^{(n)} - P_S\right) - \mu_2\left(\sum_{k=1}^{K}\sum_{l=1}^{L}\sum_{n=1}^{N}p_{2,k,l}^{(n)} - P_R\right) \\ &-\sum_{k=1}^{K}\sigma_k\left(T_{1,k}+T_{2,k}-T\right) - \sum_{n=1}^{N}\delta_1^{(n)}\left(\sum_{k=1}^{K}\xi_{1,k}^{(n)} - 1\right) \\ &-\sum_{n=1}^{N}\delta_2^{(n)}\left(\sum_{k=1}^{K}\sum_{l=1}^{L}\xi_{2,k,l}^{(n)} - 1\right) \end{aligned} \quad (10)$$

where constant B/N is omitted in order to simplify the derivation. We define $\lambda_k\,(k=1,...,K)$, μ_1, μ_2, $\sigma_k\,(k=1,...,K)$, $\delta_1^{(n)}$, and $\delta_2^{(n)}\,(n=1,...,N)$ as Lagrange multipliers.

Each subcarrier is allocated to one node exclusively. In the first hop, the subcarrier *n* is allocated to relay node k^* when k^* satisfies the following rule, which can be derived from the KKT condition [Nam07], as:

$$k^* = \arg\max_{k} \left[\left(\frac{1}{2} - \lambda_k \right)^2 \log_2 \left(1 + p_{1,k}^{(n)} \gamma_{1,k}^{(n)} \right) - \mu_1 p_{1,k}^{(n)} \right], \quad (\forall k) \tag{11}$$

In the second hop, the subcarrier n is allocated to the relay-destination pair $\{k', l'\}$ which satisfies the same rule, as:

$$\{k', l'\} = \arg\max_{\{k,l\}} \left[\left(\frac{1}{2} + \lambda_k \right)^2 \log_2 \left(1 + p_{2,k,l}^{(n)} \gamma_{2,k,l}^{(n)} \right) - \mu_2 p_{2,k,l}^{(n)} \right], (\forall k, l) \tag{12}$$

Note that k^* and k' may denote different relays. Assuming λ_{k^*}, $\lambda_{k'}$, μ_1, and μ_2 are the optimal Lagrange multipliers, the resulting optimal power and time slot durations allocated to relay k^* and k' can be derived as:

$$p_{1,k^*}^{(n)} = \left[\frac{\left(\frac{1}{2} - \lambda_{k^*} \right)^2}{\mu_1 \ln 2} - \frac{1}{\gamma_{1,k^*}^{(n)}} \right]^+ \tag{13}$$

$$p_{2,k,l'}^{(n)} = \left[\frac{\left(\frac{1}{2} + \lambda_{k'} \right)^2}{\mu_2 \ln 2} - \frac{1}{\gamma_{2,k',l'}^{(n)}} \right]^+ \tag{14}$$

$$T_{1,k^*} = \left(\frac{1}{2} - \lambda_{k^*} \right) T \tag{15}$$

$$T_{2,k'} = \left(\frac{1}{2} + \lambda_{k'} \right) T \tag{16}$$

where $[\bullet]^+ = \max\{\bullet, 0\}$. Note that when symmetric time allocation [Salem10] is applied, *i.e.*, $T_{1,k^*} = T_{2,k'} = T/2$, the optimal power allocation results are the same as (13)-(16), except that $\left(1/2 - \lambda_{k^*}\right)^2$ in (13) and $\left(1/2 + \lambda_{k'}\right)^2$ in (14) are replaced with $\left(1/4 - \lambda_{k^*}\right)$ and $\left(1/4 + \lambda_{k'}\right)$, respectively. Hence, the proposed ARA algorithm has a similar complexity to the SRA algorithm. In addition, the value of these multipliers is the limited information which needs to be fed back from the relay nodes to BS. The amount of feedback information increases linearly with the number of relays in the cell.

The following $(K+2)$ equations are to be solved to obtain the optimal Lagrangian multipliers:

$$P_S - \sum_{k=1}^{K} \sum_{n=1}^{N} \xi_{1,k}^{(n)} \left[\frac{\left(\frac{1}{2} - \lambda_k \right)^2}{\mu_1 \ln 2} - \frac{1}{\gamma_{1,k}^{(n)}} \right]^+ = 0 \tag{17}$$

$$P_R - \sum_{k=1}^{K} \sum_{l=1}^{L} \sum_{n=1}^{N} \xi_{2,k,l}^{(n)} \left[\frac{\left(\frac{1}{2} + \lambda_k \right)^2}{\mu_2 \ln 2} - \frac{1}{\gamma_{2,k,l}^{(n)}} \right]^+ = 0 \tag{18}$$

$$\begin{aligned} &\left(1 + 2\lambda_k\right) \sum_{l=1}^{L} \sum_{n=1}^{N} \xi_{2,k,l}^{(n)} \left[\log_2 \left(\frac{\left(\frac{1}{2} + \lambda_k \right)^2}{\mu_2 \ln 2} \gamma_{2,k,l}^{(n)} \right) \right]^+ \\ &- \left(1 - 2\lambda_k\right) \sum_{n=1}^{N} \xi_{1,k}^{(n)} \left[\log_2 \left(\frac{\left(\frac{1}{2} - \lambda_k \right)^2}{\mu_1 \ln 2} \gamma_{1,k}^{(n)} \right) \right]^+ = 0, (\forall k) \end{aligned} \tag{19}$$

Various search algorithms such as the sub-gradient algorithm [Nam07], can be applied to solve (17)-(19).

Simulation Results

In our simulations, we use N=256 subcarriers, K=4 relays, and L=8 destinations (except for FIGs. 3 and 4). All relays are uniformly distributed within a band region of width w=20 m, as shown in FIG. 1. The channel is modelled as six independent Rayleigh fading paths with the root-mean square (RMS) delay spread of 0.5 μs. We apply the Type-D (Roof-to-Roof) PL model [Hart06] in the first hop with PL attenuation coefficient $A_1 = 2.05 f_c^{2.6} \times 10^{-26}$, where $f_c = 5$ GHz is the central carrier frequency. The PL exponent for the first hop is $\tau_1 = 4.5$. Applying the Type-E (Roof-to-Ground) PL model [Hart06] in the second hop, we have the PL attenuation coefficient as $A_2 = 38.4$ dB, and PL exponent as $\tau_2 = 3.5$, respectively. The total bandwidth is B=50 MHz for the downlink [Seidel08], and the single-sided power spectral density of AWGN is $N_0 = -173.8$ dBm/Hz. The total transmit power is $P_T = 40$ dBm, and the total powers of P_S and P_R are 38 dBm. The total transmit time is T=5 ms. In all Figures except FIG. 5, the distance between source and destinations is fixed at $d_{SD} = 2000$ m. The distance between the source and the mid circle of the relays d_{SR} varies between 600 m and 1800 m. The ARA approach in [Zhou11] is implemented for comparison, where the whole cellular system is divided into K=4 single-relay sub-systems.

FIG. 2 illustrates the impact of the locations of relays on the maximum achievable system throughput. The proposed ARA scheme outperforms the single-relay based ARA scheme in [Zhou11] and the multi-relay

based SRA scheme [Salem10], especially when the relays are relatively close to the source. For instance, when $d_{SR} = 600$ m, the performance of the proposed ARA is around 46.9% and 70.5% higher than that of single-relay ARA and SRA, respectively. It can also be observed that the performances of proposed ARA and single-relay ARA have the same tendency of increasing when d_{SR} is in the range of 600 m to 1800 m.

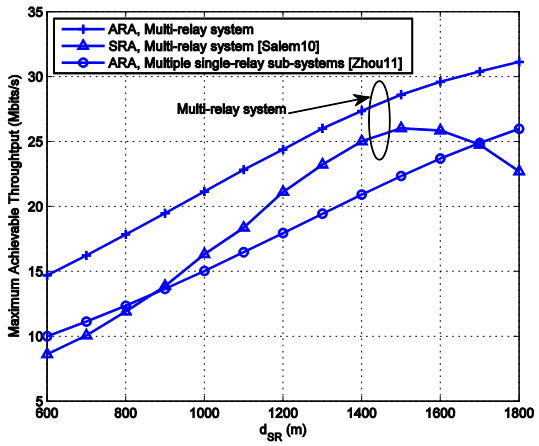


FIG. 2 IMPACT OF DISTANCE BETWEEN THE SOURCE AND THE MID CIRCLE OF RELAYS ON MAXIMUM ACHIEVABLE THROUGHPUT, WITH $K = 4$ RELAYS AND $L = 8$ DESTINATIONS

When the d_{SR} further increases, the performance of SRA deteriorates rapidly, as the symmetric time allocation cannot compensate for the performance imbalance caused by different levels of PL attenuation in the two hops.

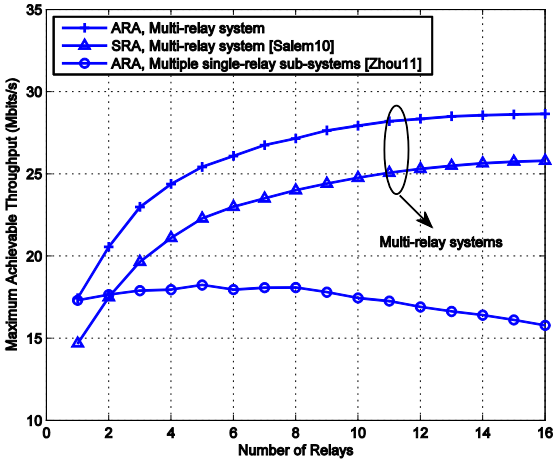


FIG. 3 IMPACT OF THE NUMBER OF RELAYS ON MAXIMUM ACHIEVABLE THROUGHPUT, WITH $L = 16$ DESTINATIONS

In FIG. 3, the impact of the number of relays on the maximum achievable throughput is illustrated, where $d_{SR} = 1200$ m. The number of destinations is fixed as L=16. It is shown that when the number of relays increases from 1 to 16, the system throughput of proposed ARA is enlarged by 65.6% due to multi-relay diversity, while the throughput of the single-relay based ARA scheme [Zhou11] decreases by 9.6% due to lack of multi-relay diversity. When the number of relays is 1, the proposed ARA scheme reduces to the method in [Zhou11].

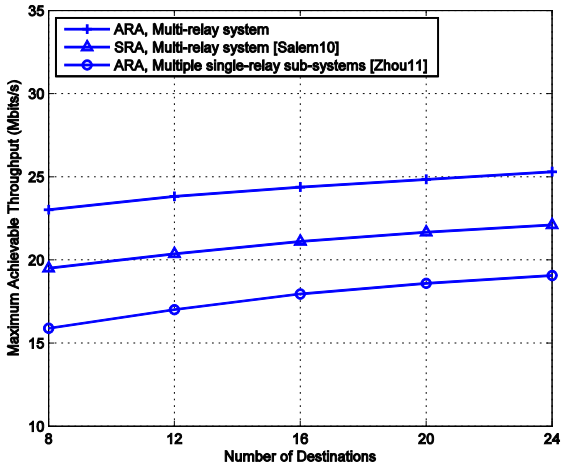


FIG. 4 IMPACT OF THE NUMBER OF DESTINATIONS ON MAXIMUM ACHIEVABLE THROUGHPUT, WITH $K = 4$ RELAYS

With the same configuration as FIG. 3, FIG. 4 illustrates the impact of the number of destinations on the maximum achievable system throughput, with K=4 relays. It can be observed that when the number of destinations increases from 8 to 24, the system throughput has little increase, which demonstrates that the multi-user diversity has less impact on performance compared with multi-time diversity and multi-relay diversity, as shown in FIGs. 2 and 3.

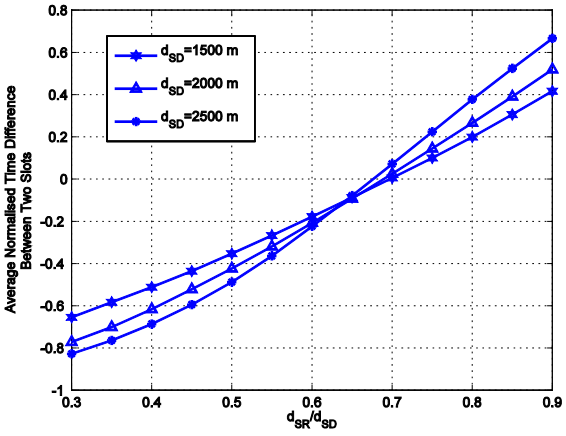


FIG. 4 IMPACT OF NORMALISED DISTANCE BETWEEN SOURCE AND DESTINATIONS ON THE AVERAGE TIME DIFFERENCE BETWEEN TWO SLOTS, WITH $K = 4$ RELAYS AND $L = 8$ DESTINATIONS

Defining d_{SR}/d_{SD} as the ratio of the distance between

source and the mid circle of the relays band to the total distance between source and destinations, we demonstrate the impact of d_{SR}/d_{SD} on ΔT_{ave} in FIG. 5, where $\Delta T_{ave} = \sum_{k=1}^{K} E\left\{\left(T_{1,k} - T_{2,k}\right)/T\right\}/K$ denotes the average normalised time difference between two hops. It can be observed that when relays are closer to source than destinations, the optimal time duration in the first hop is smaller than that in the second hop. For instance, when $d_{SR}/d_{SD} = 0.3$, $\Delta T_{ave} = -0.78$ for $d_{SD} = 2000$ m, which indicates the time duration allocated in the second hop can combat the higher level of PL attenuation caused by longer distance to achieve the same throughputs with the first hop. It is also observed that the longer the distances between source and destinations are, the steeper the curve is. Therefore, when destinations are further away from source, asymmetric time allocation plays a more important role in system performance. It can be deduced that symmetric time allocation may be a preferable alternative at $d_{SR}/d_{SD} = 0.65 \sim 0.7$ (irrespective of the value of d_{SD}), due to easier synchronisation.

Conclusions

In this paper, we have investigated ARA for DF dual-hop multi-relay OFDMA cellular systems in the downlink. Our proposed ARA scheme performs joint time, power and subcarrier allocation to achieve an enhanced degree of freedom from the multi-time, multi-relay, and multi-user diversities. Simulation results show that asymmetric time allocation plays an important role in system performance. As a result, the proposed ARA scheme can outperform the SRA scheme [Salem10] significantly due to multi-time diversity, at a similar complexity, especially when relays are relatively close to source. Also, when the distances between source and destinations are relatively large, the impact of asymmetric allocation is more significant. Furthermore, the proposed optimal multi-relay ARA scheme outperforms the single-relay based ARA scheme in [Zhou11], thanks to the multi-relay diversity. Simulation results also show that the multi-user diversity plays a less significant role than the multi-relay and multi-time diversities in the system performance.

Appendix – Proof of Convexity of the Objective Function (9)

It can be proved that the objective function (9) is a convex optimisation problem, by proving the convexity of each TC_k in (9). For the purpose of simplicity in the proof, we define $q_{1,k}^{(n)} = p_{1,k}^{(n)}/\xi_{1,k}^{(n)}$ and $q_{2,k,l}^{(n)} = p_{2,k,l}^{(n)}/\xi_{2,k,l}^{(n)}$, where $\xi_{1,k}^{(n)}$ and $\xi_{2,k,l}^{(n)}$ are relaxed to continuous variables which are in the interval of [0,1]. Using (1), (2), (5) and $T_{1,k}C_{1,k} = T_{2,k}C_{2,k}$, we can derive

$$\frac{T_{1,k}}{T_{2,k}} = \frac{C_{2,k}}{C_{1,k}} = \frac{\sum_{l=1}^{L}\sum_{n=1}^{N}\xi_{2,k,l}^{(n)}\log_2\left(1+q_{2,k,l}^{(n)}\gamma_{2,k,l}^{(n)}\right)}{\sum_{n=1}^{N}\xi_{1,k}^{(n)}\log_2\left(1+q_{1,k}^{(n)}\gamma_{1,k}^{(n)}\right)} \quad (20)$$

And using (1), (2), (20) and $TC_k = \left(T_{1,k}C_{1,k} + T_{2,k}C_{2,k}\right)/2$, we have

$$\begin{aligned} C_k &= \frac{1}{2}\left(\frac{T_{1,k}}{T}C_{1,k} + \frac{T_{2,k}}{T}C_{2,k}\right) \\ &= \frac{1}{2}\left(\frac{C_{2,k}}{C_{1,k}+C_{2,k}}C_{1,k} + \frac{C_{1,k}}{C_{1,k}+C_{2,k}}C_{2,k}\right) \\ &= \frac{1}{C_{1,k}} + \frac{1}{C_{2,k}} \end{aligned} \quad (21)$$

Hence, the maximum achievable system throughput via relay *k* can be expressed as

$$C_k = \frac{B}{N}\left[\frac{1}{\sum_{n=1}^{N}\xi_{1,k}^{(n)}\log_2\left(1+q_{1,k}^{(n)}\gamma_{1,k}^{(n)}\right)} + \frac{1}{\sum_{l=1}^{L}\sum_{n=1}^{N}\xi_{2,k,l}^{(n)}\log_2\left(1+q_{2,k,l}^{(n)}\gamma_{2,k,l}^{(n)}\right)}\right] \quad (22)$$

The concavity of $\sum_{n=1}^{N}\xi_{1,k}^{(n)}\log_2\left(1+q_{1,k}^{(n)}\gamma_{1,k}^{(n)}\right)$ and $\sum_{l=1}^{L}\sum_{n=1}^{N}\xi_{2,k,l}^{(n)}\log_2\left(1+q_{2,k,l}^{(n)}\gamma_{2,k,l}^{(n)}\right)$ are proved first. Note that $\sum_{n=1}^{N}\xi_{1,k}^{(n)}\log_2\left(1+q_{1,k}^{(n)}\gamma_{1,k}^{(n)}\right)$ can be decomposed into N components, and the nth component is $\xi_{1,k}^{(n)}\log_2\left(1+q_{1,k}^{(n)}\gamma_{1,k}^{(n)}\right)$. Defining a function $f\left(\xi_{1,k}^{(n)}, p_{1,k}^{(n)}\right) = \xi_{1,k}^{(n)}\log_2\left(1+p_{1,k}^{(n)}\gamma_{1,k}^{(n)}/\xi_{1,k}^{(n)}\right)$, the Hessian matrix of $f\left(\xi_{1,k}^{(n)}, p_{1,k}^{(n)}\right)$ can be written as

$$\begin{aligned} \mathbf{H}(f) &= \begin{bmatrix} \frac{\partial^2 f}{\partial\left(\xi_{1,k}^{(n)}\right)^2} & \frac{\partial^2 f}{\partial\xi_{1,k}^{(n)}\partial p_{1,k}^{(n)}} \\ \frac{\partial^2 f}{\partial p_{1,k}^{(n)}\partial\xi_{1,k}^{(n)}} & \frac{\partial^2 f}{\partial\left(p_{1,k}^{(n)}\right)^2} \end{bmatrix} \\ &= \frac{1}{\ln 2}\begin{bmatrix} -\frac{\left(p_{1,k}^{(n)}\right)^2}{\xi_{1,k}^{(n)}\left(\xi_{1,k}^{(n)}+p_{1,k}^{(n)}\right)^2} & \frac{p_{1,k}^{(n)}}{\left(\xi_{1,k}^{(n)}+p_{1,k}^{(n)}\right)^2} \\ \frac{p_{1,k}^{(n)}}{\left(\xi_{1,k}^{(n)}+p_{1,k}^{(n)}\right)^2} & -\frac{\xi_{1,k}^{(n)}}{\left(\xi_{1,k}^{(n)}+p_{1,k}^{(n)}\right)^2} \end{bmatrix} \end{aligned} \quad (23)$$

Defining $\mathbf{r}=\left[r_1, r_2\right]$ as a non-zero real valued number vector, we have

$$\mathbf{rH}\left(f\right)\mathbf{r}^T = -\frac{1}{\ln 2\left(\xi_{1,k}^{(n)}+p_{1,k}^{(n)}\right)^2}\left(\frac{p_{1,k}^{(n)}}{\sqrt{\xi_{1,k}^{(n)}}}r_1-\sqrt{\xi_{1,k}^{(n)}}r_2\right)^2 \quad (24)$$

It is obvious that when $0<\xi_{1,k}^{(n)}<1$ and $p_{1,k}^{(n)}>0$, $\mathbf{rH}(f)\mathbf{r}^T<0$ always holds, which means that $\mathbf{H}(f)$ is a negative definite matrix. Thus, it can be concluded that $f\left(\xi_{1,k}^{(n)}, p_{1,k}^{(n)}\right)$ is concave. Accordingly, $\sum_{n=1}^{N}\xi_{1,k}^{(n)}\log_2\left(1+p_{1,k}^{(n)}\gamma_{1,k}^{(n)}\right)$ is concave. Similarly, $\sum_{l=1}^{L}\sum_{n=1}^{N}\xi_{2,k,l}^{(n)}\log_2\left(1+p_{2,k,l}^{(n)}\gamma_{2,k,l}^{(n)}\right)$ is concave. Therefore, (22) is convex, and (9) is convex objective function, which means that the globe optimal solution to (9) can be obtained, as given by (13)-(16).

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